KEY UNDERSTANDING 4

We can calculate one measurement from others using relationships between quantities.

In the everyday world, many of the measurements we use have not been obtained directly but have been derived from other measurements by undertaking calculations. This may involve:

choosing and using an operation, such as:

- adding the quantities shown on each of the packs to decide how much mince there is in the freezer
- weighing ourselves on the bathroom scales, weighing ourselves holding the cat, and find the difference to find the weight of the cat
- measuring the thickness of a thousand sheets (a ream) of paper and dividing the measurement by one thousand to measure the thickness of a sheet of paper

choosing and using a rate or scale, such as:

- finding the volume of a container by finding the mass of the water it holds, and using the fact that water weighs one gram per cubic centimetre
- estimating the time it will take to travel between two towns using the anticipated speed (a rate) and the distance
- using a measurement on the map, and the scale factor of 1000 to estimate a real distance

choosing and using a formula, such as:

- finding the area of a rectangle by measuring the lengths of two adjacent sides and multiplying the two measurements
- using a baby's weight and a formula relating the amount of medicine needed to body weight to work out the right dose of medicine.

Students should learn to recognise when a calculation would help solve a practical measurement problem, work out which calculations to do and do them correctly.

Working out whether and when a calculation is possible involves thoughtfulness and judgment. For example, students may have learned through activities such as those described in Key Understanding 1 that the area of a rectangle can be found by multiplying its length by its width. Confronted with the problem of finding the area of a garden bed, they then have to decide whether they can use this rule or formula. *Is the garden a rectangle? Can we check? Is it close enough for my purposes? If not, can I break the region up into smaller rectangles that I can find the area of?* and so on.

If the students decide that a particular formula may be used, they will need to decide what component measurements are required and apply the formula correctly. Applying the formula correctly is not simply a matter of computational skill (which is dealt with in Number: Calculate), it involves first checking that the units of measurement are appropriate and doing any needed conversions.

Students who have achieved Level 3 can choose operations in relatively straightforward situations. For example, they may add the lengths of the sides of a shape to find its perimeter, or subtract a TV program starting time from its finishing time to work out if the three-hour videotape is long enough.

Students who have achieved Level 4 can carry out calculations with measurements involving decimals. They use the relationship between quantities to work out one quantity from another and will make some of their own measurement short cuts. For example, they might multiply the length of one side of a regular polygon by five to get the perimeter or find the volume of a prism composed of cubes by multiplying the number of layers by the number in each layer.

Students who have achieved Level 5 can choose and use straightforward formulae with which they are familiar, including working out what measurements they need to make in order to use the formula and ensuring that the units are consistent.

SAMPLE LEARNING ACTIVITIES

Beginning ✔

Incidental

Let students see and hear your calculations when you combine measurements for a purpose. For example, while planning the assembly, say: We will allow about 2 minutes for the speech by Ms James and 3 minutes each for the two songs, so that is 8 minutes so far. Or, say: We are going to need two cups of starch for each batch and there will be three batches, so we will need six cups of starch.

Does It Work?

After students have used a common unit to measure the length of various paper tapes or ribbons, ask them to predict the total length if they were to be arranged in a long line. Ask: What would we need to do to work out the total length? Would your calculator be helpful? Have them check their calculations by laying out tapes (ribbons) end-to-end and measuring the total. Repeat for other combinations of lengths. (Link to Direct Measure, Key Understanding 3.)

Class Party

Invite students to solve problems that involve combining quantities. For example, say: In preparing for the class party, Mrs Williams poured one cup of cordial into the jug and then added nine cups of water. Ask: How much drink did she make? How do you know it is that much? What if she wanted to make double that amount? How many cups of cordial and how many cups of water would she need? How did you work it out? (Link to ***First Steps in Mathematics: Number***, Understand Fractional Numbers, Key Understanding 7, and Understand Operations, Key Understanding 3.)

Cooking

In cooking activities, involve students in planning the quantities and writing out new recipes. (e.g. doubling the ingredients for a cake, making enough dough for two scones for each student, calculating the ingredients for home-made lemonade from a recipe for four) (Link to ***First Steps in Mathematics: Number***, Understand Operations, Key Understanding 3.)

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Excursions

Have students help plan excursions. For example, invite them to work out when they will return to school. Ask: How long will we spend at the destination? How long will it take to get there? How long will it take to get home? How long will we spend having lunch or snacks? Help students see how periods of time are combined and related to the starting times to enable them to tell parents when they will return to school. (Link to Direct Measure, Key Understanding 6.)

Friezes

Have students work out the length of a border for the pin board. Encourage them to decide on a suitable unit, count how many on each side and attach that number to each side. Ask: What do you notice about the top and bottom and side measurements? Invite them to work out the total length without re-measuring. Ask: Can you see a short cut for measuring another pin board?

Frame a Picture

Invite students to work out how much card they need to frame their art. Ask: What measurements will be needed? How will the corners go together? Will this make a difference to the measurements? How can we work out how much wood (card) will be needed altogether? It comes in lengths of 1 metre, 1.5 metres and 2 metres. Which would be the best to use? Have students measure and construct their frames. (Link to Direct Measure, Key Understanding 4.)

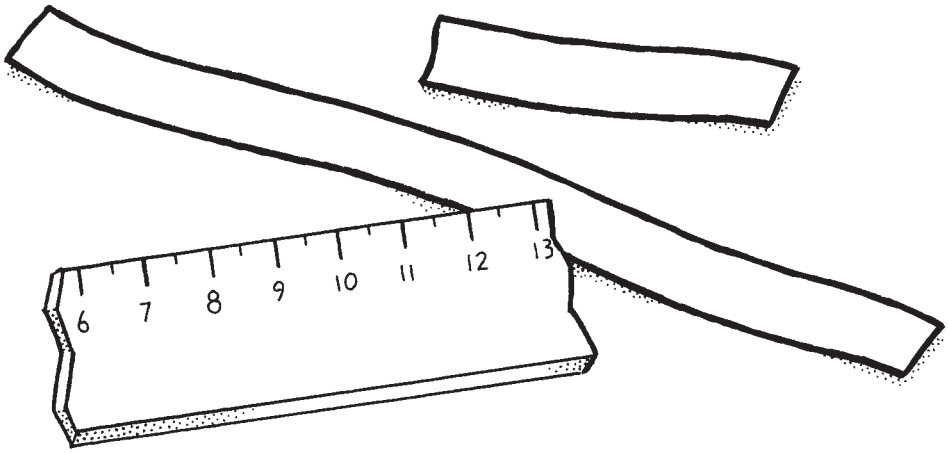
Overcoming Limitations

Have students overcome limitations in the measurement range of equipment. For example, give students kitchen scales that weigh up to 250 grams. Ask them to find the mass of a bag of flour (which weighs more than 250 grams). Then, ask them to find about 400 grams of tomatoes to use in a sauce. Ask: What calculations did you need to do? How can you check your results a different way? (Link to Direct Measure, Key Understanding 4.)

Middle ✔✔

Broken Ruler

Ask students to measure and calculate to overcome inaccuracies in equipment. For example, give students parts of a broken ruler (they should have different parts) or paper tape marked like a broken ruler. Ask them to find the length of both shorter and longer items and say how they were able to measure in centimetres. Ask: How can you work out the length without having to count each centimetre gap? What calculations can you use? Which measurements involved more calculations? Why? Does it matter which part of the ruler you have? Should you get the same result? (See Direct Measure, Key Understanding 4.)



Combined Mass

Ask students to find objects that have a combined mass of 1 kilogram (combined length of 1 metre). Ask: How did you do this? Did you need to use any calculations? How did you find out how much the last object had to weigh (measure)? Was it difficult to find something that was just right? (Link to Estimate, Key Understanding 2.)

Weighing Awkward Objects

Have students use bathroom scales to find the mass of objects that can be difficult to weigh without special scales (e.g. small animals, bags of fruit or vegetables, a packed suitcase) by weighing themselves, weighing themselves together with the object, then calculating the difference. Ask: Why does this method work? How accurate is it likely to be? Could we weigh a very tiny kitten using this method? Why? Why not? (Link to Direct Measure, Key Understanding 4, and Understand Units, Key Understanding 5.)

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

Recycled Cans

Have students investigate how much the school receives for recycled aluminium cans. Invite them to use this information to calculate the total mass of the cans collected by their class so far, the total mass of cans collected in the school, the amount of money the school will receive and a prediction of how much money the school will receive by the end of the year. Ask: How can you work out the total mass when you can't fit all of the cans on the scales? What do you need to know to work out how much money the school should receive? (Link to ***First Steps in Mathematics: Number***, Understand Operations, Key Understanding 3.)

Overcoming Limitations

Invite students to find a way to measure things that are too small for the accuracy of the equipment available (e.g. the thickness of a single piece of paper using only their ruler, the mass of a grain of rice using kitchen scales, the volume of a drop of water using a measuring cylinder). Compare the methods and the operations used for each measure. Ask: Which were the quickest and easiest to carry out? Did the different methods produce different answers? Why did this happen? How could the range be reduced? (See Understand Units, Key Understanding 5; link to ***First Steps in Mathematics: Number***, Understand Operations, Key Understanding 4.)

It Needs Fixing

Ask students to calculate to address inaccuracies in equipment. For example, say: Our tape measure has stretched, so when I use it to measure an object that my stretched tape shows is 1 metre long, I know that the real length of the object is 1.2 centimetres longer than 1 metre. Ask:

- What would the real length of the room be if my stretched tape measure shows it as 4 metres long?
- What would be the real length of a chair that my tape measure shows as 50 centimetres?
- What would be the real length of my desk that my tape measure shows as 1.50 metres?

Have students share the calculations they used to work out the real lengths. (See Direct Measure, Key Understanding 4.)

KU 4

Later ✔✔✔

Dripping Tap

Have students measure the quantity of water wasted from a dripping tap in one day. Ask: Is there any way we could work it out without leaving the bucket under the tap all day? How could we use this information to work out how much water would be wasted in a week? Invite students to work it out. Then, ask: Which measurements did you need to make? What calculations did you need to do? How could you adjust the time measurement to make the calculations easier? (Link Understand Units, Key Understanding 6.)

Oil Spills

Have students find the areas of a range of irregular regions (not given on grid paper), such as the aerial view of oil spills. (See Middle Sample Learning Activity 'Oil Spills' in Key Understanding 1). Limit the materials students can use to paper tiles, ruler, pencil and calculator. Invite students to explain how they worked it out. Ask: How can you use the length-by-width rule to avoid counting all the squares? (Link to Direct Measure, Key Understanding 3; see Sample Lesson 4, page 64.)

Using Perimeter

Extend 'Oil Spills' by having students test the incorrect hypothesis that you can use the perimeter of a region to work out the area. For example, present the following conflict situation. Say: Someone in the other class found a very quick and easy method to work out the area of a diagram of an oil slick. They taped string around the edge of the shape then cut and joined the ends of the string. They then made the string into a rectangle, and multiplied the height and the width measures of the rectangle to work out the area. Ask: Do you think this method would give you a measure of the area? How do you know? How could you test this? How would you convince the student from the other class?

Using a Formula

Have students decide when it would make sense to use a particular calculation or formula and when it wouldn't. For example, present the following problems and ask: Would it make sense to multiply 4 by 10 to get an answer? Why? Why not?

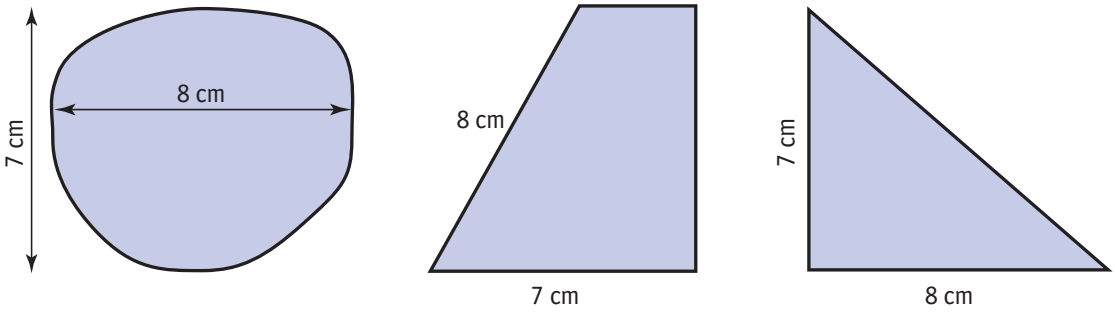
- A man can run a kilometre in 4 minutes. How long would it take him to run 10 kilometres?
- A kilogram of apples costs $4. How much would it cost for 10 kilograms?

Present the following questions and ask: Could you sensibly use the length-by-width rule to answer the following questions? Why? Why not?

- The school oval measures 70 metres by 50 metres. What is its area?

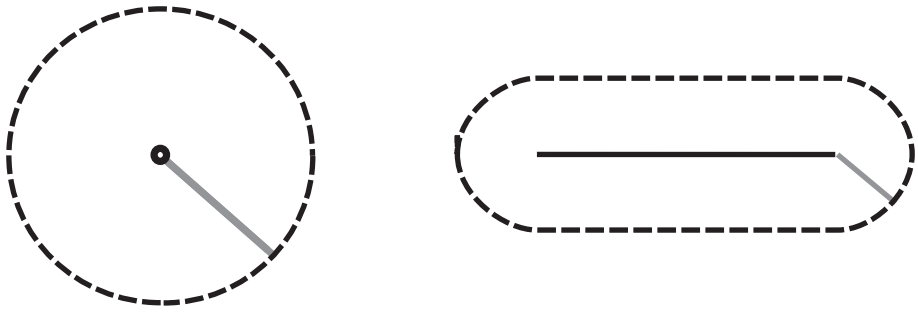
- A rectangular paddock measures 70 metres long and 50 metres wide. What is its area?
- A rectangular park is 70 metres long and 50 metres wide. How much fencing will be needed to enclose it?

Show students the following shapes and ask: why wouldn't it make sense to multiply 7 times 8 to find the area of these shapes? (Link to ***First Steps in Mathematics: Number***, Understand Operations, Key Understanding 9.)



Grazing Areas

Say: There are two goats. The first goat is tethered by a lead to a stake in the ground. The second goat is tethered by a lead half as long as the first goat's lead to a sliding rail that is double the length of the first goat's lead. Invite students to use a compass and ruler to draw representations of the two feed areas. Give students cubes, paper tiles, or pencil and ruler to work it out and then ask: How did you work out which animal has the larger grazing area? Encourage students to discuss and justify the method they chose. Draw out the strategies that were the quickest and easiest to use. Ask: How do you know these strategies are as accurate as counting all the squares? (Link to Key Understanding 1; see Direct Measure, Key Understanding 2, and link to Direct Measure, Key Understandings 3 and 5.)



goat 1 **goat 2**